APPROXIMATE CALCULATION OF FALL VELOCITY OF SPHERICAL PARTICLE IN GENERALIZED NEWTONIAN FLUID IN CREEPING REGION

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The paper gives a relation of the Rabinowitsch–Mooney type between the consistency variables τ_s and D_s which has been suggested, together with the force balance, for approximate calculation of the fall velocity of spherical particle in generalized Newtonian fluid. Its applicability to solutions characterized by the Ellis flow model has been experimentally proved.

In refs[1 – 3] we dealt with the calculation of the pressure drop connected with a flow of compressible and noncompressible Newtonian Fluid (NF) and Generalized Newtonian Fluid (GNF) through a fixed randomly packed bed of both spherical and nonspherical particles. The difference from the former way of solving this problem consisted in that the resistance of layer was divided into the frictional and shape components. This led to introduction of a dimensionless quantity given by the ratio of the shape and frictional resistances of spherical particles which is called the resistance criterion. Using the approximate presumption of agreement between the distribution of stress during the flows of an NF and a GNF through a fixed bed of particles, we applied the Rabinowitsch–Mooney equation (for a pipe) to the calculation of pressure drop.

The aim of this work is to experimentally verify whether or not the approximate presumption of agreement between the distribution of stress in an NF and a GNF can be used – together with the thereto corresponding force balance and equation of the Rabinowitsch–Mooney type – for an approximate calculation of fall velocity of spherical particle in GNF in creeping region.

THEORETICAL

The solution by Stokes[4] of the flow past a spherical particle can be written in the form

$$D_s = 2\,u_{ch}/l_{ch} = \tau_s/\mu\,, \qquad (1)$$

$$f_t = f_f + f_s \equiv f_f(1 + \psi) . \tag{2}$$

Here D_s and $\tau_s \equiv f_f$ are the consistency variables of the problem solved, the characteristic velocity of system, u_{ch}, is the fall velocity, the characteristic linear dimension of system, l_{ch}, is the diameter d of the particle, f_t is the total resistance, f_f is the frictional resistance, and f_s is the shape resistance of spherical particle referred to its surface area, and μ is the Newtonian dynamic viscosity, the resistance criterion being $\psi = 1/2$.

With the approximate presumption of agreement between distribution of stress during flow of an NF and a GNF past a spherical particle, an equation of Rabinowitsch–Mooney type was derived in ref.[5] in the following form:

$$D_s \equiv 2\,u_{ch}/l_{ch} = (\tau_s^{1/2}/2)\int_0^{\tau_s} \tau^{-3/2}\, D(\tau)\, d\tau , \tag{3}$$

where $D(\tau)$ represents the dependence of shear rate D on the shear stress τ whose course is given generally by the flow curve of GNF or by the respective flow model.

From the presumption introduced it further follows that in the force balance (Eq. (*2*)) the resistance criterion has the value of 1/2 again. Hence, for a fall of a particle we get the following relation for the τ_s quantity from the force balance (Eq. (*2*)):

$$\tau_s = (\rho_s - \rho)\, g\, d/9 , \tag{4}$$

where ρ_s and ρ are densities of the particle and the liquid, respectively, and g is the gravitational acceleration.

As it follows from Eq. (*3*) the problem can easily be solved for the flow models which explicitly express the dependence of the shear rate D on the shear stress τ. From among the models of this type which also respect the Newtonian behaviour of GNF in the region of $\tau \rightarrow 0$, the model by Ellis is the most simple.

$$D = (\tau/\eta_0)\,[1 + (\tau/\tau_{1/2})^{\alpha - 1}] , \tag{5}$$

where η_0, $\tau_{1/2}$, and α are its parameters. For this model the solution of Eq. (*3*) leads to the relation:

$$D_s = (\tau_s/\eta_0)\,[1 + (\tau_s/\tau_{1/2})^{\alpha - 1}/(2\,\alpha - 1)] . \tag{6}$$

The treatment of the problem of the fall of particles also involves the way in which the effect of walls is taken into account. As it follows from the solution by Stokes the fall velocity of the particle depends on the way of distribution of stress, and in an apparatus limited by walls it is generally impossible to expect an agreement between the stress distribution in an NF and a GNF. Hence it cannot generally be expected that the way of involvement of the effect of walls which is used for Newtonian systems will be satisfactory for non-Newtonian fluids as well.

However, if this way of involvement of the effect of walls agrees with experimental results obtained for non-Newtonian fluids at least up to a certain value of the ratio $(d/D_k)_{max}$, where D_k is the diameter of cylindrical column, then – in the region $d/D_k \in [0; (d/D_k)_{max}]$ – a satisfactory agreement can be expected between the stress distribution in a Newtonian and a non-Newtonian fluids, which agreement is necessary for the application of the method suggested for determination of fall velocity of spherical particle.

EXPERIMENTAL

The fall velocity of glass, steel, and lead spheres was measured in aqueous solutions of hydroxyethylcellulose Cellosize QP-40 (Union Carbide Corp., U.S.A.), Natrosol 250 MR and Natrosol 250 H (Hercules Powder Comp., Holland), in a solution of polyethylene oxide WSR 301 (Union Carbide Corp. U.S.A.), and in a solution of polyacrylamide Separan AP-45 (Dow Chemical Corp., Switzerland). The fall velocities were measured in cylindrical columns of 20, 8, 4, and 2 cm diameters.

The physical properties of the particles used are given in Table I. The given values of densities ρ_s were determined pycnometrically; the given diameter d of particles was calculated from the weight of 50 particles and their density ρ_s using the formula for calculation of volume of a spherical particle.

The flow curves were determined by measurements carried out on a rotary viscometer Reotest 2. The following procedure was adopted for estimation of the zero-shear viscosity η_0: the values of fall velocities u_g measured in the columns of the two greatest diameters, where a satisfactory validity of the way of involvement of effect of walls used for Newtonian fluids could be anticipated, were divided by the Faxén correction factor[6] for the effect of walls in the following form:

$$F_u \equiv u_g/u_{ch} = 1 - 2.104\,(d/D_k) + 2.09\,(d/D_k)^3 . \tag{7}$$

The velocity values u_{ch} thus calculated were introduced, together with the known values of the quantities $l_{ch} \equiv d$ and τ_s according to Eq. (4), into Eq. (1) in which the quantity μ has – for GNF – the meaning

TABLE I
Physical properties of spherical particles used

Particle No.	$d \equiv l_{ch}$, mm	ρ_s, kg m^{-3}	Particle No.	$d \equiv l_{ch}$, mm	ρ_s, kg m^{-3}
1	1.465	2 506	8	3.178	7 834
2	1.923	2 527	9	3.973	7 826
3	2.782	2 504	10	4.752	7 808
4	3.457	2 867	11	1.973	11 090
5	4.117	2 596	12	2.851	11 190
6	0.823	7 646	13	3.902	11 160
7	1.992	7 877			

of the so-called effective viscosity of the system. The values of effective viscosity μ were plotted against the u_{ch} values, the zero-shear viscosity η_0 being estimated by extrapolation of the μ values for the condition of $u_{ch} \rightarrow 0$. The remaining two parameters of the Ellis model were determined by the method by Turian[7].

The values of parameters of the Ellis model and the other characteristics of the fluids used are given in Tables II and III. The density values ρ given there and the parameters of the Ellis flow model were estimated at the temperature equal to the arithmetic mean of the minimum and the maximum temperatures used for the measurements of the fall velocity of the particle in the given solution. The temperatures of solutions varied within the limits of ±0.9 °C about the mean temperature.

TABLE II
Polymer solutions used

Solution No.	Composition	Density, ρ kg m^{-3}	Parameters of Ellis model		
			η_0, Pa s	$\tau_{1/2}$, Pa	α
1	2.4% Cellosize QP-40	1 002	0.85	110.8	1.65
2	2.5% Cellosize QP-40	1 003	0.98	71.2	1.86
3	1.3% Natrosol 250 MR	1 001	2.25	8.76	1.89
4	1.5% Natrosol 250 MR	1 003	3.30	11.64	2.16
5	1.0% Natrosol 250 H	1 005	3.00	3.99	1.87
6	1.2% Natrosol 250 H	1 002	6.50	9.94	2.37
7	1.3% Natrosol 250 H	1 003	10.25	11.54	2.14
8	1.3% Polyox WSR 301	1 001	3.30	4.39	2.55
9	0.5% Separan AP-45	999	9.00	1.32	3.22

TABLE III
Characteristics of solutions used and results of measurements

Solution No.	Range of D, s^{-1}	E, s	n	$(d/D_k)_{max}$	δ	δ_{num} [a]
1	0.2 – 100	0.005	32	0.206	0.042	0.067
2	2.7 – 140	0.012	33	0.238	0.035	0.049
3	0.2 – 50	0.229	36	0.206	0.060	0.065
4	0.2 – 50	0.329	40	0.206	0.039	0.066
5	0.2 – 100	0.654	32	0.206	0.091	0.137
6	1.5 – 40	0.896	33	0.238	0.055	0.098
7	0.13 – 20	1.013	36	0.238	0.207	0.196
8	0.2 – 100	1.165	30	0.238	0.503	0.181
9	0.2 – 100	15.14	24	0.206	0.480	0.257

[a] Numerical solution by Hopke and Slattery[9].

The applicability of the method suggested for estimating fall velocity was verified using 296 experimental values for different solutions and particles. The experiments were carried out in the range of values of the Reynolds number $Re \in (5 \cdot 10^{-5}; 5.4 \cdot 10^{-1})$ defined by the relation $Re = 2\,\rho u_{ch}^2/\tau_s$, which after substitution of τ_s with the use of Eq. (*1*) assumes the form used for Newtonian fluids in literature. The maximum value of ratio d/D_k measured was 0.24; the corresponding value of the Faxén correction factor[6] for the effect of walls is $F_u = 0.54$.

RESULTS AND DISCUSSION

The values of the consistency variable τ_s and of the characteristic velocity u_{ch} were calculated using Eqs (*4*) and (*6*), respectively. The value u_{ch} was multiplied by the corresponding value of the Faxén correction factor[6] for the effect of walls, F_u, using Eq. (*7*). Then the value of relative deviation was estimated for such corrected and experimental values of fall velocities

$$\delta_i = u_{g,\,exp}/u_{ch,\,corr} - 1 \qquad (8)$$

and the values of mean relative deviations were calculated for the individual solutions too (Table III):

$$\delta = 1/n \sum_{i=1}^{n} |\delta_i| \,. \qquad (9)$$

From the table it is obvious that the largest value of the mean relative deviation is exhibited by the solutions of Polyox and Separan (Nos 8 and 9). Here the applied way of involvement of effect of walls was unsatisfactory, too, because the value of relative deviation δ_i for a given particle depended upon the d/D_k ratio.

A part of experimental results in the form of relative deviation δ_i for the solution No. 7, exhibiting the second largest value of mean relative deviation δ, is given in Table IV.

TABLE IV
Comparison of experimental and corrected values of fall velocities with application of relative deviation δ_i for solution No. 7 (1.3% Natrosol 250 H)

D_k, cm	Relative deviation δ_i for particle No.		
	5	9	13
20	0.046	0.191	0.316
8	0.033	0.204	0.360
4	0.094	0.227	0.372
2	0.203	0.324	0.449

The particles given there have almost identical diameters but differ in their densities (Nos 5 glass, 9 steel, 10 lead). From the table it can be seen that for this solution the magnitude of the relative deviation δ_i depends on the density of particles, the way of involvement of effect of walls being unsatisfactory again.

With regard to these findings it seems useful – for the solution types investigated – to suitably delimit the region of satisfactory validity of Eq. (*6*).

The measure of deformation of a continuum depends on the stresses existing therein and on its material (rheological) properties. Hence it seems useful to introduce some characteristic quantity independent of the magnitude of stress and involving the parameters of the respective flow model. For the solution types investigated the $E = (\alpha - 1)\,\eta_0/\tau_{1/2}$ turned out to be suitable; its values are given in Table III.

From the table it follows that the suggested method of estimation of the fall velocity is unsatisfactory for $E > 0.9$ s. Therefore, the results for solutions Nos 7 – 9 will not be considered.

The results for the solutions Nos 1 – 6 were further arranged according to the diameters of the columns used (Table V). From the values of mean relative deviations δ given it is obvious that for the column of the smallest diameter ($D_k = 2$ cm) the mean relative deviation δ is the greatest, whereas for the remaining columns the effect of their diameter on the magnitude of deviation is insignificant.

If the results for the column of the smallest diameter are neglected, then the value of mean relative deviation δ has the value of 0.047 for the remaining set of 149 experiments limited by the value of ratio $(d/D_k)_{max} \approx 0.12$, which corresponds to the value of the Faxén correction factor $F_u \approx 0.75$.

Chhabra et al.[8] suggested a set of Eqs (*10*), (*11*) for calculation of the terminal fall velocities of spherical particle in creeping region in GNFs characterized by the Ellis flow model:

$$X = C_D\,Re_0/24 \qquad (10)$$

$$X = [1 + 0.50\,El^{1.65}(\alpha - 1)]\,. \qquad (11)$$

TABLE V
Comparison of values of mean relative deviations δ for all columns used and solutions Nos 1 – 6

D_k, cm	n	δ
20	35	0.045
8	57	0.051
4	57	0.044
2	57	0.070

The values of numerical coefficients of Eq. (11) were obtained experimentally. In Eqs (*10*) and (*11*) the drag coefficient is $C_D = 8 f_c/(\rho\, u_{ch}^2)$, the Reynolds number is $Re_0 = u_{ch}\, d\, \rho/\eta_0$, and the Ellis number is $El = 2^{1/2}\, \eta_0\, u_{ch}/(\tau_{1/2} d)$. As it is $\tau_s/\tau_{1/2} = 2^{1/2} X\, El$, and $D_s\, \eta_0/\tau_{1/2} = 2^{1/2} El$, the relation (*6*) can be modified to give Eq. (*10*) which can be compared with the relations by Chhabra et al.[8] as well as with the results of numerical solution by Hopke and Slattery[9] which provides the so-called upper and lower limits of the quantity X, the arithmetic mean of the both limits being considered to be the correct result.

This comparison for the value of the parameter $\alpha = 1.88$ is presented in Fig. 1 which also gives experimental results for the values of $\alpha = 1.87$ (solution No. 5) and $\alpha = 1.89$ (solution No. 3). From the figure it can be seen that, in the whole range of the values of El number given, the agreement between the original solution and the experimental results is the best one. Moreover it follows from the figure that the solution given in this paper is "located" between the results following from the numerical treatment and empirical relations by Chhabra et al.[8] in about two thirds of the given range of El. A qualitatively identical result is also obtained from the comparison carried out for other values of the α parameter.

A full comparison of the results of numerical solution with experiment using the mean relative deviation δ according to Eq. (*9*) is given in Table III. From this table it follows that for the solutions Nos 1 – 6 the value of mean relative deviation δ estimated by us is lower than that using the results of numerical treatment, whereas for the solutions Nos 7 – 9 (for which the method suggested by us for estimation of fall velocity is unsatisfactory) the opposite is true.

With application of the numerical treatment the result for the so-called upper and lower limits of the X quantity depends on the choice of the streem function type, and from the character of the problem it does not at all follow that the arithmetic mean of the upper and lower limits of X quantity should be the correct result. With regard to the

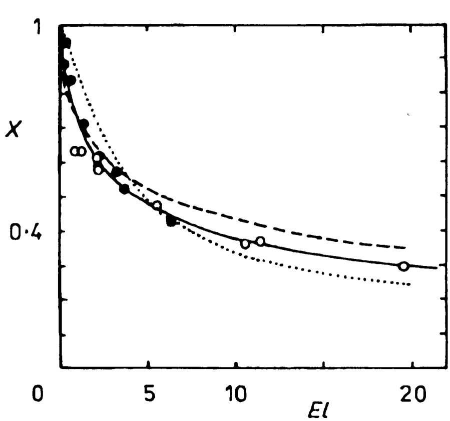

FIG. 1
Comparison of approximate solution for the Ellis flow model with relations given in literature for the value of parameter $\alpha = 1.88$ and experimental results: ——— the solution given in this paper, – – – – Hopke and Slattery[9], ... Chhabra et al.[8], our experiments ○ $\alpha = 1.87$, ● $\alpha = 1.89$

fact that the quantity X is inversely proportional to the fall velocity of the particle, we could equally well consider the arithmetic mean of the fall velocities corresponding to the upper and lower limits of the X quantity, and a priori it is impossible to say which of the two ways is more correct.

Beside its simplicity and unambiguity the method suggested by us has the advantage in being extendable also to the region where the Reynolds number makes itself felt (by adopting the results of theoretical part of ref.[2]) and to the fall of nonspherical particles in GNF (by adopting the theoretical part of ref.[3]). At the same time, these tasks require fulfilling of the basic condition of geometrical similarity and (after involving the effect of walls into the value of fall velocities) respect the fact that the problem is analogous to the solution by Stokes, i.e., it is axisymmetrical. Therefrom it follows that the set of Eqs (*1*) – (*3*) together with some of the dependences $C_D = C_D(Re)$ given in literature for the fall of spherical particle in a Newtonian fluid will be applicable to predictions of fall velocity of spherical particle in GNFs only up to a certain critical value of the Reynolds number Re from which the so-called secondary motion becomes significant. In such case the trajectory of the falling particle is not a straight line but a helix. Also it can be expected that the above-mentioned critical value of Re will depend on the type of the model fluid, which is supported by experimental results obtained in our laboratory in the context of studies of problems of fluidized bed[10].

CONCLUSIONS

The dividing of resistance into a friction and a shape components together with the presumed agreement between the distribution of stress during flow of GNF past a spherical particle enabled a simple and sufficiently accurate way of calculation of the fall velocity of spherical particle in GNF in creeping region.

Its suitability for the investigated types of solutions characterized by the Ellis flow model was limited by the value of the quantity $E < 0.9$ s and up to the value of ratio $(d/D_k)_{max} \approx 0.12$ which is, according to Eq. (*7*), connected with the value of the correction factor for the effect of walls $F_u \approx 0.75$.

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SYMBOLS

C_D	drag coefficient
D	shear rate, s^{-1}
D_k	diameter of cylindrical column, m
D_s	consistency variable, s^{-1}
d	diameter of spherical particle, m
E	characteristic quantity of Ellis flow model, s
El	Ellis number

F_u	correction factor for effect of walls
f_t	total resistance of spherical particle referred to its surface area, Pa
f_s	shape resistance of spherical particle referred to its surface area, Pa
f_f	friction resistance of spherical particle referred to its surface area, Pa
g	gravitational acceleration, m s^{-1}
l_{ch}	characterictic linear dimension of system, m
n	number of experiments for a given liquid
Re	Reynolds number
Re_0	Reynolds number for Ellis liquid
$u_{g,\,exp}$	fall velocity of particle, m s^{-1}
u_{ch}	characteristic velocity of system, m s^{-1}
$u_{ch,\,corr}$	corrected characteristic velocity of system, m s^{-1}
X	dimensionless quantity of Eq. (*10*)
α	parameter of Ellis flow model
δ	mean relative deviation
δ_i	relative deviation
η_0	parameter of Ellis flow model, Pa s
μ	dynamic or effective viscosity, Pa s
ρ	density of liquid, kg m^{-3}
ρ_s	density of particle, kg m^{-3}
τ	shear stress, Pa
$\tau_{1/2}$	parameter of Ellis flow model, Pa
τ_s	consistency variable, Pa
ψ	resistance criterion

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